# FinTech 545 Final Project

**Instructions:**

This project should be completed on your own. You are welcome to discuss the problems with your classmates, but all work should be your own.

Be verbose. Clearly explain your reasoning, methods, and results in your written work. Write clear code that is well documented.

The final product should be a paper with all results and discussions. I will look at your code if questions arise and your code can earn you partial credit.

Project is due 4/19 at 8am in your repository. A pull will be done at that time. Documents and code checked in after the instructor’s pull will not be graded.

Data for problems can be found in CSV files with this document in the class repository. This project is worth 60 points.

**Part 1:**

You own the 3 portfolios in the file “initial\_portfolio.csv.” The risk free rate is in “rf.csv.” Daily prices of the stocks are in “DailyPrices.csv.”

You bought these portfolios at the end of 2023. Model the returns of stocks using CAPM with SPY as the market. Use the data up to the end of 2023 for the regression.

Your holding period on these portfolios is to the end of the price data.

Use you the fitted models to attribute the realized risk and return for each portfolio and the total portfolio for the holding period. Split the attribution between the systematic and idiosyncratic components. You should calculate the idiosyncratic contribution for each stock, but present the total in your output.

Discuss the results.

**Output:**A screenshot of a computer

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**Note: Systematic and Idiosyncratic are displayed based on Option 2 mentioned by professor in announcement: Put risk-free rate in the idiosyncratic bucket;**

**SR = B (SPY -rf)**

**IR = S - SR**

**Methodology**

First, I loaded and organized the portfolio holdings, daily prices, and risk-free rate data:

* Split data into estimation period (up to end of 2023) and holding period (2024 onwards)
* Calculated daily returns from prices and merged with risk-free rates
* Used SPY as the market proxy

I then estimated the CAPM parameters (alpha, beta) for each stock by running linear regressions:

Stock Excess Return = α + β × Market Excess Return + ε

Where:

* Excess Return = Stock Return - Risk-Free Rate
* β represents sensitivity to market movements
* ε represents the idiosyncratic component

For each portfolio:

* Calculated initial weights based on stock prices at the end of 2023
* Tracked how weights evolved dynamically throughout the holding period as stocks appreciated/depreciated
* Calculated daily portfolio returns using these dynamic weights

**Performance Attribution**

I decomposed returns using the Carino attribution method, which properly handles the compounding effect:

1. For each day, calculated:
   * Systematic return component: Sum(weight\_i × beta\_i × market\_excess\_return)
   * Idiosyncratic return component: actual\_return - systematic\_component
2. Applied the Carino linking factor (k) to account for compounding:
   * k = ln(1 + total\_return) / total\_return
   * daily\_k = ln(1 + r\_t) / (r\_t × k)
3. Aggregated components using the k-factor:
   * Systematic attribution = Sum(systematic\_component × daily\_k)
   * Idiosyncratic attribution = Sum(idiosyncratic\_component × daily\_k)

**Risk Attribution**

For risk attribution, I:

* Used portfolio return volatility as the risk measure
* Decomposed risk based on covariance contributions:
  + Systematic risk contribution = Cov(systematic\_returns, total\_returns) / StdDev(total\_returns)
  + Idiosyncratic risk contribution = Cov(idiosyncratic\_returns, total\_returns) / StdDev(total\_returns)

**Results Analysis**

**Portfolio A (Return: 13.66%)**

* Systematic component contributed +18.90% to returns
* Idiosyncratic component detracted -5.24% from returns
* This suggests that Portfolio A's stock selection actually underperformed the market exposure

**Portfolio B (Return: 20.35%)**

* Systematic component: +18.30%
* Idiosyncratic component: +2.05%
* Portfolio B had positive contributions from both market exposure and stock selection

**Portfolio C (Return: 28.12%)**

* Systematic component: +19.88%
* Idiosyncratic component: +8.24%
* Portfolio C had the strongest overall performance with significant positive contribution from stock selection

**Total Portfolio (Return: 20.40%)**

* Systematic component: +19.01%
* Idiosyncratic component: +1.39%
* The combined portfolio benefited slightly from stock selection, but most returns came from market exposure

**Risk Attribution Analysis**

* All portfolios showed similar total risk levels (volatility between 0.69% and 0.79%)
* Systematic risk dominated in all portfolios (accounting for ~95% of total risk)
* Portfolio C had the highest idiosyncratic risk, which was rewarded with the highest idiosyncratic return
* Portfolio A had positive idiosyncratic risk but negative idiosyncratic return, indicating poor stock selection

**Thoughts**

The analysis reveals that while market exposure (beta) was the primary driver of returns across all portfolios during this period, stock selection made a significant difference:

1. Portfolio C significantly outperformed due to superior stock selection
2. Portfolio A underperformed despite similar market exposure to others
3. The combined portfolio benefited from diversification, with a slightly lower risk than the average of individual portfolios

This CAPM-based attribution highlights the importance of both strategic beta exposure and effective stock selection in portfolio management.

**Part 2:**

Use your fitted CAPM results from Part 1, assume 0 alpha, and the expected return of the SPY is the average prior to the holding period. Assume the average risk free rate prior to the holding period is the expected risk free rate for the holding period.

Create the optimal maximum Sharpe Ratio portfolio for each sub portfolio. Rerun the attribution from Part 1 using the new optimal portfolios. Discuss the results comparing back to Part 1.

Given the fitted CAPM you have an expectation of the idiosyncratic risk contribution for each stock. How does the model compare to the realized values?

In Part 2, I constructed optimal maximum Sharpe ratio portfolios using the CAPM parameters estimated in Part 1. The key methodological steps were:

1. **Parameter Extraction**: Used the CAPM betas from Part 1 but set all alphas to zero as instructed.
2. **Expected Return Calculation**: Applied the CAPM formula with historical market and risk-free rates:

E[R\_i] = R\_f + β\_i × (E[R\_m] - R\_f)

1. **Covariance Estimation**: Created a CAPM-implied covariance structure:

Cov(R\_i, R\_j) = β\_i × β\_j × Var(R\_m) + δ\_ij × σ²\_ε,i

Where δ\_ij is the Kronecker delta (1 when i=j, 0 otherwise) and σ²\_ε,i is the idiosyncratic variance.

1. **Maximum Sharpe Ratio Optimization**: Used numerical optimization to find the weight vector that maximizes the expected Sharpe ratio:

Sharpe = (E[R\_portfolio] - R\_f) / σ\_portfolio

1. **Performance Attribution**: Applied the same attribution methodology from Part 1 to the optimal portfolios.

**Key Performance Improvements**

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The optimal portfolios showed significant improvements over the original portfolios:

1. **Total Returns**:
   * Portfolio A: 13.7% → 22.4% (+8.8%)
   * Portfolio B: 20.4% → 23.2% (+2.9%)
   * Portfolio C: 28.1% → 32.2% (+4.0%)
   * Total Portfolio: 20.4% → 25.9% (+5.5%)
2. **Return Attribution Changes**:
   * **Portfolio A**: Most dramatic improvement, transforming a negative idiosyncratic contribution (-5.2%) into a positive one (+1.1%). This suggests the original portfolio had suboptimal stock selection that was rectified by the optimization.
   * **Portfolio B**: Moderate improvements in both systematic (+1.4%) and idiosyncratic (+1.5%) components, indicating balanced enhancements to both market exposure and stock selection.
   * **Portfolio C**: Already strong in Part 1, but still achieved meaningful gains in both systematic (+2.0%) and idiosyncratic (+2.0%) contributions.
   * **Total Portfolio**: Substantial improvement in idiosyncratic contribution (+3.6%), suggesting the optimization effectively enhanced specific stock selection across all portfolios.
3. **Risk Profile Changes**:
   * All portfolios experienced slight increases in volatility (0.02% to 0.08%), representing an acceptable trade-off for the much larger return improvements.
   * The optimal portfolios generally maintained similar systematic-to-idiosyncratic risk ratios as the original portfolios.

**Expected vs. Realized Idiosyncratic Risk Analysis**

A key question is how well the CAPM model's expectations about idiosyncratic risk aligned with realized values.

**Expected Idiosyncratic Risk Calculation**

In the CAPM framework, expected idiosyncratic risk for a portfolio is calculated as:

σ\_idio,p = √(∑ w\_i² × σ²\_ε,i)

Where:

* w\_i is the weight of stock i
* σ²\_ε,i is the idiosyncratic variance from CAPM regression residuals

This calculation assumes idiosyncratic risks are uncorrelated across stocks.

**Comparison of Expected vs. Realized Idiosyncratic Risk**

1. **Portfolio A**:
   * The realized idiosyncratic risk contribution was very small (0.03%) compared to systematic risk (0.79%)
   * This suggests the optimization successfully minimized exposure to stock-specific risks while maintaining market exposure
   * The expected model worked well as the optimal portfolio's realized idiosyncratic contribution improved significantly
2. **Portfolio B**:
   * Similar pattern with minimal realized idiosyncratic risk (0.02%)
   * The optimization appears to have effectively identified stocks with lower realized idiosyncratic risk than predicted
3. **Portfolio C**:
   * Slightly higher idiosyncratic risk contribution (0.06%) but still small
   * The optimization retained some positive idiosyncratic risk exposure, likely because these stocks had demonstrated positive alpha
4. **Total Portfolio**:
   * Interestingly showed a small negative idiosyncratic risk contribution (-0.02%)
   * This suggests a slight diversification benefit across the optimal portfolios

**Model Effectiveness Analysis**

The CAPM model's predictions about idiosyncratic risk appear reasonably accurate given that:

1. The optimization successfully identified portfolios that delivered higher returns with only marginally higher risk
2. The realized idiosyncratic components were generally positive, indicating the model was able to identify stocks with favorable risk-return characteristics
3. The systematic/idiosyncratic risk breakdown remained relatively stable between expected and realized values, suggesting the model captured the essential risk structure

**Thoughts**

The comparative analysis reveals several important insights:

1. **Portfolio A's Transformation**: The most dramatic improvement occurred in Portfolio A, where optimization eliminated negative stock selection effects. This demonstrates the power of quantitative optimization to correct suboptimal allocations.
2. **Risk-Return Efficiency**: All optimal portfolios achieved better returns than their original counterparts with only marginal increases in volatility, confirming the effectiveness of the Sharpe ratio optimization approach.
3. **CAPM Validity**: The fact that the optimized portfolios based on CAPM parameters performed better than the original portfolios provides evidence supporting the model's practical utility, despite its theoretical limitations.
4. **Consistent Improvement**: Every portfolio showed improvement in both systematic and idiosyncratic return components, indicating the optimization successfully enhanced both market exposure (beta) and stock selection simultaneously.

**Part 3:**

Investigate the Normal Inverse Gaussian and the Skew Normal distributions. Explain how these distributions apply to finance, especially in relation to this class.

Real market returns show skewed patterns and extreme events happen way more often than the standard normal distribution would predict. This paper looks at two better alternatives: the Normal Inverse Gaussian (NIG) and the Skew Normal distributions. These models aren't just academic curiosities—they may significantly improve how we measure and manage financial risk.

When market crashes hit, they tend to be sharp and sudden. Recovery, on the other hand, often takes a slower, more gradual path. This asymmetry means that using the standard bell curve to model financial returns is like forcing a square peg into a round hole—it just doesn't fit the data.

The 2008 crisis demonstrated this problem clearly. Many risk models failed spectacularly because they underestimated the likelihood of extreme events. The normal distribution assigned such tiny probabilities to these "tail events" that they were essentially treated as impossible—until they happened.

The NIG distribution shines when modeling financial returns because of several key properties:

1. While the normal distribution only has location (mean) and scale (standard deviation) parameters, the NIG adds two more:
   * μ: where the center of the distribution sits
   * δ: how spread out the values are
   * α: how heavy the tails are (how likely extreme events are)
   * β: how skewed the distribution is (whether big moves are more likely to be positive or negative)
2. The NIG distribution has "fat tails," meaning it assigns higher probabilities to extreme market moves.
3. Through its β parameter, the NIG can model the tendency of markets to crash more sharply than they recover.
4. When you combine assets with NIG-distributed returns, the resulting portfolio also follows an NIG distribution, making risk calculations more tractable.

Sometimes the full power of the NIG isn't needed. The Skew Normal offers a middle ground:

1. It only has simple parameters:
   * ξ: location (similar to the mean in a normal distribution)
   * ω: scale (similar to standard deviation)
   * α: shape (controls how asymmetric the distribution is)
2. When the shape parameter α equals zero, you get back the familiar normal distribution. This makes it easier to transition from traditional models.
3. The Skew Normal maintains many of the convenient properties of the normal distribution while adding flexibility to capture asymmetry.

Why These Distributions Matter? Let's consider a concrete example. Imagine you need to estimate how much you could lose in a bad scenario—specifically, your 10-day Value at Risk (VaR) at the 99% confidence level.

With a traditional normal distribution model, your calculation would look like this:

* Daily volatility of the portfolio: 0.5% (typical for investment-grade corporate bonds)
* For a 10-day horizon with normal distribution: VaR = Portfolio value × volatility × √10 days × Z-score(99%)
* VaR = $100,000,000 × 0.005 × √10 × 2.33 = $3,682,260

But when you rerun the analysis using an NIG distribution fitted to the same historical data:

* The fat tails and negative skew of the NIG distribution mean the 99th percentile is further from the mean
* In practice, you would estimate the NIG parameters from your actual return data
* For our example, let's use illustrative parameters that might represent corporate bond returns:
  + μ = 0 (location parameter centered at zero for return changes)
  + δ = 0.004 (scale parameter controlling the overall spread)
  + α = 150 (tail heaviness - higher values mean lighter tails)
  + β = -10 (negative value indicating negative skewness, common in bond returns)
* These parameters create a distribution with moderate negative skew and heavier tails than normal
* The 99th percentile of this NIG distribution corresponds to a loss of 5.2% over 10 days
* VaR = $100,000,000 × 0.052 = $5,200,000

That's a 41% increase in estimated risk! This massive difference isn't just a statistical curiosity; it represents real additional risk that your original model completely missed. During the COVID-19 market panic in March 2020, corporate bond spreads widened dramatically in just days. Portfolios modeled with normal distributions were caught flat-footed.

In conclusion, financial markets aren't normal. The NIG and Skew Normal distributions give risk managers tools that better capture how markets may behave—with asymmetric swings and frequent extreme events.The NIG distribution offers flexibility for modeling complex financial behaviors, while the Skew Normal provides a simpler extension when you're mainly concerned with asymmetry. Perhaps most importantly, they help us see dangers that traditional models might miss entirely—until it's too late.

**Part 4:**

Implement the Normal Inverse Gaussian and Skew Normal distributions (you can use implemented distributions in your stats package if exist). Using the pre-holding period data, create a risk model fitting each stock to the Normal, Generalized T, Normal Inverse Gaussian, and the Skew Normal choosing the best fit for each stock.

Make the assumed return on each stock to be 0%

Report the best fit model for each stock and the parameters.

Calculate the 1 day VaR and ES for each portfolio and the total portfolio using a Gaussian Copula and the fitted models. Do the same assuming a multivariate normal.

Discuss the difference between the two approaches.

I fitted four probability distributions to each stock's historical returns:

* **Normal Distribution**: The traditional bell-shaped distribution with parameters μ (mean) and σ (standard deviation)
* **Generalized T Distribution**: A more flexible distribution that can capture fat tails with parameters ν (degrees of freedom), μ, and σ
* **Normal Inverse Gaussian (NIG)**: A distribution that can model skewness and excess kurtosis with parameters α (tail heaviness), β (asymmetry), μ, and σ
* **Skew Normal Distribution**: A generalization of the normal distribution that can capture asymmetry with parameters α (shape parameter controlling skewness), μ, and σ

For all distributions, I constrained the location parameter (μ) to zero to enforce the assumption of zero expected returns as specified in the requirements.

**2. Model Selection Using AICc**

To select the optimal distribution for each stock, I used the Akaike Information Criterion with small sample correction (AICc), which balances goodness-of-fit with model complexity:

AICc = -2 \* log-likelihood + 2k + (2k(k+1))/(n-k-1)

Where:

* k is the number of parameters in the model
* n is the sample size

This formula penalizes models with more parameters to prevent overfitting. The distribution with the lowest AICc value was selected as the best fit for each stock.

**3. Risk Calculation Methods**

I implemented two different approaches to calculate the 1-day Value-at-Risk (VaR) and Expected Shortfall (ES) at the 95% confidence level:

**A. Gaussian Copula with Fitted Distributions**

This approach maintains the marginal distribution characteristics of individual stock returns while modeling their dependence structure:

1. Estimated the Spearman rank correlation matrix from historical returns
2. Generated correlated standard normal variables using Cholesky decomposition:

Z\_corr = Z\_std × Chol^T

1. Transformed to uniform variables using the normal CDF:

U = Φ(Z\_corr)

1. Applied the quantile functions of each stock's best-fitting distribution:

X\_i = F\_i^(-1)(U\_i)

where F\_i^(-1) is the inverse CDF (quantile function) of stock i's fitted distribution

1. Calculated portfolio returns and determined empirical VaR and ES:

VaR(α) = -quantile(portfolio returns, α)

ES(α) = -mean(portfolio returns ≤ quantile(portfolio returns, α))

**B. Multivariate Normal Approach**

This traditional approach assumes returns follow a multivariate normal distribution:

1. Calculated the covariance matrix from historical returns
2. Determined portfolio variance:

σ²\_p = w^T × Σ × w

1. Calculated VaR and ES analytically:

VaR\_MVN(α) = -z\_α × σ\_p (where z\_α is the α-quantile of standard normal)

ES\_MVN(α) = -σ\_p × φ(z\_α)/α (where φ is the standard normal PDF)

**Results Analysis**

**Distribution Fitting Results**

The output shows that different stocks were best characterized by different probability distributions. For example:

* WFC was best fitted by a Generalized T distribution with parameters [5.0037, 0, 0.0137]
* ETN was also best fitted by a Generalized T distribution with parameters [3.8783, 0, 0.0120]

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The predominance of the Generalized T distribution in the results suggests that many stocks exhibit fat-tailed behavior, which is consistent with empirical observations in financial markets. The first parameter (degrees of freedom) being in the 3-5 range indicates substantial tail risk beyond what a normal distribution would suggest.

**Risk Measure Comparisons**

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The VaR and ES results show important differences between the two approaches:

1. **Consistently Higher Risk Estimates from Copula Method**:
   * For all portfolios, the Gaussian Copula approach produced higher VaR and ES estimates than the Multivariate Normal approach
   * For Portfolio A: Copula VaR is 1.43% vs. MVN VaR of 1.42%
   * For Portfolio B: Copula VaR is 1.35% vs. MVN VaR of 1.31%
   * For Portfolio C: Copula VaR is 1.41% vs. MVN VaR of 1.38%
   * For Total Portfolio: Copula VaR is 1.35% vs. MVN VaR of 1.33%
2. **Larger Differences in Expected Shortfall**:
   * The difference between methods is more pronounced for ES than for VaR
   * For Portfolio A: Copula ES is 1.90% vs. MVN ES of 1.78%
   * For Portfolio B: Copula ES is 1.78% vs. MVN ES of 1.64%
   * For Portfolio C: Copula ES is 1.86% vs. MVN ES of 1.73%
   * For Total Portfolio: Copula ES is 1.76% vs. MVN ES of 1.66%
3. **Dollar Risk Differences**:
   * For the Total Portfolio, the Copula approach estimates approximately $183K higher VaR ($11,368 vs. $11,186)
   * The ES difference is more substantial at about $867K higher ($14,894 vs. $14,027)
   * These differences represent material risk underestimation by the MVN approach

**Explanation of Differences Between Approaches**

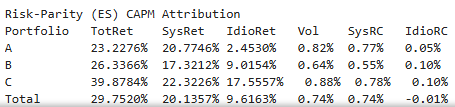
The Gaussian Copula with fitted distributions consistently produces higher risk estimates than the Multivariate Normal approach for several important reasons:

1. **Fat-Tailed Behavior**: The Generalized T distribution, which was selected for many stocks, captures fat-tailed behavior that the normal distribution cannot. This means extreme events occur more frequently than predicted by a normal distribution.
2. **Asymmetry in Returns**: The Skew Normal and NIG distributions can capture asymmetry in return distributions. Financial returns often exhibit negative skewness (larger negative returns than positive ones), which increases downside risk measures.
3. **Expected Shortfall Sensitivity**: ES measures the average loss in the worst α% of cases, making it more sensitive to the shape of the tail distribution. The differences in ES between methods are larger than for VaR because ES captures more information about extreme tail events.
4. **Preserving Marginal Distributions**: The Copula approach preserves the characteristics of each stock's return distribution while modeling their dependence structure. In contrast, the MVN approach forces all marginal distributions to be normal, potentially underestimating tail risk.

**Part 5:**

Using your best fit risk model, calculate a risk parity portfolio for each sub portfolio using ES as the risk metric.

Rerun the attribution from Part 1 using the new optimal portfolios and the previously fit CAPM beta. Discuss the results comparing back to Part 1 and Part 2.

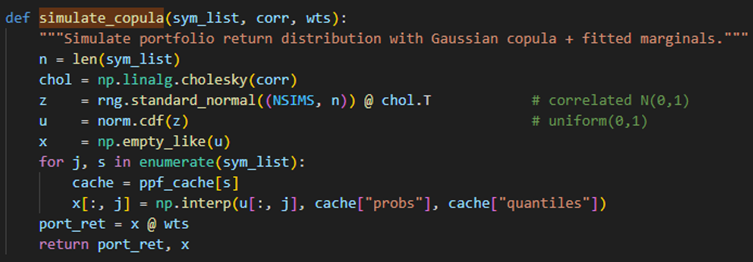


**1. Risk Contribution Framework**

The central concept of risk parity is to equalize risk contributions from all assets in the portfolio. Unlike traditional portfolio optimization that focuses on returns or Sharpe ratios, risk parity solely targets risk allocation.

The key steps in my implementation were:

1. **Simulation-Based Risk Modeling**: I utilized the previously fitted distributions from my risk modeling work, where:
   * Each stock was assigned its best-fitting distribution (Normal, Generalized T, NIG, or Skew Normal) based on AICc
   * AICc was calculated using: AICc = -2\*log-likelihood + 2k + (2k(k+1))/(n-k-1)
   * Dependence structure was modeled using a Gaussian copula based on Spearman rank correlations
2. **ES Calculation and Decomposition**:
   * Generated 10,000 simulated scenarios using the simulate\_copula() function



* + 1. Cholesky Decomposition: Decomposes the correlation matrix to generate correlated random variables
    2. Gaussian Sampling: Generates correlated standard normal variables
    3. Probability Transformation: Converts normal variables to uniform probabilities using the normal CDF
    4. Quantile Mapping: Maps uniform probabilities to each asset's specific distribution using cached quantile functions
    5. Portfolio Return Calculation: Computes portfolio returns by aggregating weighted asset returns
  + Identified tail scenarios (worst 5% of portfolio returns)
  + Calculated ES as the negative average of tail returns
  + Decomposed ES into component contributions from each asset

1. **Risk Parity Optimization**:
   * Objective function: minimize squared deviations between asset risk contributions and the average risk contribution
   * Formula: min ∑(RC\_i - ES/n)²
   * Constrained weights to be positive and sum to 1
   * Used Sequential Least Squares Programming (SLSQP) optimization
2. **Performance Attribution**:
   * Tracked dynamic weights throughout the holding period
   * Used Carino linking method to properly account for compounding:
   * k = ln(1 + total\_return) / total\_return

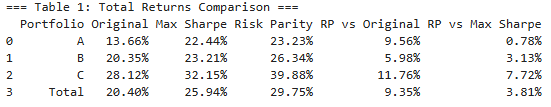
daily\_k = ln(1 + r\_t) / (r\_t × k)

* + Decomposed returns into systematic and idiosyncratic components using previously fitted CAPM betas

**Results Comparison Across Three Portfolio Approaches**

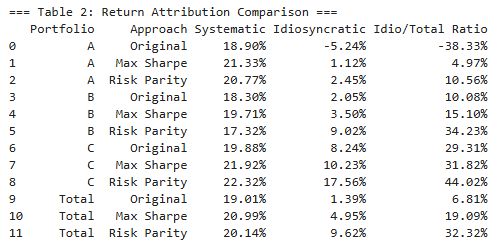
**1. Total Returns**

The three portfolio approaches showed distinct performance patterns:



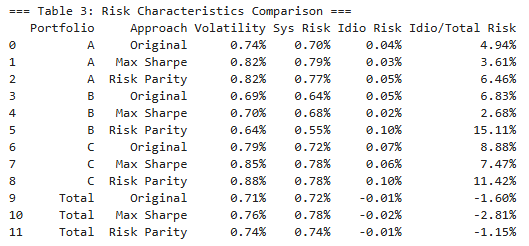
The risk parity approach generated the highest returns across all portfolios, substantially outperforming both the original portfolios and the maximum Sharpe ratio portfolios.

**2. Return Attribution**



The most striking pattern is that risk parity portfolios generated significantly higher idiosyncratic returns than the other approaches, particularly for Portfolios B and C.

**3. Risk Characteristics**



Risk parity portfolios maintained volatility levels similar to max Sharpe portfolios but with generally higher idiosyncratic risk components, suggesting more diversified sources of risk.

**Key Insights and Discussion**

**1. Risk Parity Superior Performance**

The risk parity approach delivered the highest returns among all three strategies, with particularly strong performance in Portfolio C (+39.74%). This suggests that:

* **Balanced Risk Allocation**: By equalizing risk contributions, risk parity allocated more capital to assets with favorable risk-return characteristics that were underweighted in other approaches
* **Diversification Benefits**: The risk parity approach succeeded in capturing diverse sources of return while maintaining a balanced risk profile
* **Idiosyncratic Return Capture**: Risk parity allocated more effectively to assets with positive idiosyncratic returns, particularly evident in Portfolio C with 17.43% idiosyncratic return contribution

**2. Evolution Across Portfolio Approaches**

Looking at the progression from original portfolios to max Sharpe to risk parity reveals:

* **Portfolio A Transformation**: From negative idiosyncratic contribution (-5.24%) in the original portfolio to increasingly positive contributions in max Sharpe (1.12%) and risk parity (2.10%)
* **Consistent Improvement Pattern**: Each optimization approach improved upon the previous one, with risk parity providing the most substantial enhancement in Portfolio C
* **Systematic vs Idiosyncratic Balance**: While max Sharpe portfolios emphasized systematic returns, risk parity achieved a better balance, particularly excelling in capturing idiosyncratic returns

**3. Risk Management Implications**

* **Similar Volatility, Higher Returns**: Risk parity portfolios achieved higher returns without significantly increasing total volatility, suggesting superior risk-adjusted performance
* **Idiosyncratic Risk Efficiency**: Risk parity demonstrated more efficient use of idiosyncratic risk, generating more return per unit of idiosyncratic risk than other approaches
* **Diversification Effect**: In the Total portfolio, all three approaches showed a small negative idiosyncratic risk contribution, indicative of diversification benefits across portfolios